Manipulating Expressions

Name_____________________
Classwork Book
Math 7: Miss Zgoda
Lesson 1: Combining Like Terms

Example 1: Any Order, Any Grouping Property with Addition

a. Rewrite $5x + 3x$ and $5x - 3x$ by combining like terms.
   Write the original expressions and expand each term using addition. What are the new expressions equivalent to?

b. Find the sum of $2x + 1$ and $5x$.

c. Find the sum of $-3a + 2$ and $5a - 3$.

d. $3(2x)$

e. $4 \cdot 2 \cdot z$

f. $3(2x) + 4y(5)$
Exercises: Simplify each expression by combining like terms.

1. $-6k + 7k$

2. $-2x + 11 + 6x$

3. $12r + 5 + 3r - 5$

4. $n + 4 - 9 - 5n$

5. Find the sum of $-4p + 5$ and $3p - 10$

6. Find the sum of $-3x + 6y - 2z$ and $9x + 10z - 10y$

7. $4y(5)$

8. $3(2x) + 4y(5) + 4 \cdot 2 \cdot z$

9. Alexander says that $3x + 4y$ is equivalent to $(3)(4) + xy$ because of the Commutative and Associative Properties. Is he correct? Why or why not?
Relevant Vocabulary

**Variable (description):** A *variable* is a symbol (such as a letter) that represents a ________________, (it is a placeholder for it).

**Numerical Expression (description):** A *numerical expression* is a number, or it is any combination of sums, differences, products, or divisions of numbers that ________________ to a number.

**Value of a Numerical Expression:** The *value of a numerical expression* is the number found by evaluating the ________________.

**Expression (description):** An *expression* is a numerical expression, or it is the result of ________________ some (or all) of the numbers in a numerical expression with variables.

**Equivalent Expressions:** Two expressions are *equivalent* if both expressions ________________ the same number for every substitution of numbers into all the letters in both expressions.

**An Expression in Expanded Form:** An expression that is written as sums (and/or differences) of products whose factors are numbers, variables, or variables raised to whole number powers is said to be in *expanded form*. A single number, variable, or a single product of numbers and/or variables is also considered to be in expanded form. Examples of expressions in expanded form include: $324$, $3x$, $5x + 3 - 40$, $x + 2x + 3x$, etc.

**Term (description):** Each summand of an expression in ________________ form is called a *term*. For example, the expression $2x + 3x + 5$ consists of 3 terms: $2x$, $3x$, and $5$.

**Coefficient of the Term (description):** The number found by ________________ just the numbers in a term together. For example, given the product $2 \cdot x \cdot 4$, its equivalent term is $8x$. The number $8$ is called the ________________ of the term $8x$.

**An Expression in Standard Form:** An expression in expanded form with all its like terms collected is said to be in *standard form*. For example, $2x + 3x + 5$ is an expression written in expanded form; however, to be written in ________________ form, the like terms $2x$ and $3x$ must be _________________. The equivalent expression $5x + 5$ is written in standard form.
## Problem Set

For problems 1–6, write equivalent expressions by combining like terms.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$3a + 5a$</td>
<td>2.</td>
</tr>
<tr>
<td>4.</td>
<td>$-7h - 8h + 20h$</td>
<td>5.</td>
</tr>
<tr>
<td>7.</td>
<td>$8g + 8 - 4g$</td>
<td>8.</td>
</tr>
<tr>
<td>10.</td>
<td>$2x + 3y - 4x$</td>
<td>11.</td>
</tr>
</tbody>
</table>

Jack and Jill were asked to find the sum of $2x + 1$ and $5x$.

13. Jack got the expression $7x + 1$, then wrote his answer as $1 + 7x$. Is his answer an equivalent expression? How do you know?

14. Jill also got the expression $7x + 1$, then wrote her answer as $1x + 7$. Is her expression an equivalent expression? How do you know?
Lesson 2: Adding and Subtracting Expressions

Example 1: Subtracting Expressions

a. Subtract: \((4c + 9) - (3c + 2)\).

b. Subtract: \((3x + 5y - 4) - (4x + 11)\).

Example 2: Combining Expressions

a. Find the sum.
\((5a + 3b - 6c) + (2a - 4b + 13c)\)

b. Find the difference.
\((2x + 3y - 4) - (5x + 2)\)
Exercises: Subtract each of the following
1. \((3n + 15) - (n + 3)\)

2. \((2b - 3) - (4b - 1)\)

3. \((2x^2 + 4x - 8) - (5x^2 - 6x + 10)\)

Exercises: Combine expressions.
1. \((5p^3 - 3) + (2p^2 - 3p^3)\)

2. \((3a^2 + 1) + (4 + 2a^2)\)

3. \((4n - 3n^3) - (3n^3 + 4n)\)

4. \((5a + 4) - (5a + 3)\)

5. \((3 - 6n^5 - 8n^4) + (-6n^4 - 3n - 8n^5)\)
## Problem Set

1. Write each expression in standard form.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>a.</strong></td>
<td>(3x + (2 - 4x))</td>
<td><strong>b.</strong> (3x + (-2 + 4x))</td>
</tr>
<tr>
<td><strong>d.</strong></td>
<td>(-3x + (-2 - 4x))</td>
<td><strong>e.</strong> (3x - (-2 + 4x))</td>
</tr>
</tbody>
</table>

2. Write each expression in standard form.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>a.</strong></td>
<td>(4y - (3 + y))</td>
<td><strong>b.</strong> ((6c - 4) - (c - 3))</td>
</tr>
<tr>
<td><strong>d.</strong></td>
<td>((2g + 9h - 5) - (6g - 4h + 2))</td>
<td></td>
</tr>
</tbody>
</table>
Lesson 3: Equivalent Expression Models

Algebra Tiles are great tools to use for understanding equivalent expressions and equations.

Here are the basics:
There are three types of tiles; large squares, small squares and rectangles.

a. Small squares are worth 1 unit, it is a 1 by ______ square.

b. Rectangles are worth $x$ units, it is a 1 by ______ rectangle.

c. Big squares are worth $x^2$ units, it is an $x$ by ______ square.

Each tile has two sides. Depending on your tiles, one side represents a negative value. The other side is positive.

The expression, $4x$, would be modeled like this:

```
+  
+  
+  
+  
```

To represent $4x + 3$, you need to add ______ units to the previous model.

```
+  +  
+  +  
+  +  
+  +  
```

It is important to understand that any tile and its opposite are equal to zero and can be removed.

```
+  -  = 0
```

Operations with Algebra tiles are easy to do as well.

a. You are allowed to add or subtract to any collection of tiles by putting extra tiles in or taking them away.

b. You can multiply an expression creating rectangular area models.

c. You can divide by splitting into equal sized groups.
Example 1
Model the expressions and sketch below.

d. $2x - 5$

e. $-4x + 2$

f. $-x^2 + 3x + 4$

g. $3(x - 2)$

h. $2(-2x + 1)$

i. $-2(-2x + 1)$

j. $(x + 2)(x + 3)$
Problem Set

1. Answer each part below.
   a. Write two equivalent expressions that represent the rectangle below.

   \[ \begin{array}{c}
   \text{3} \\
   2a \\
   5 \\
   \end{array} \]

   b. Verify informally that the two equations are equivalent by substituting 3 for \( a \).

2. Use a rectangular array to write the products as sums.
   a. \( 2(x + 10) \)

   b. \( 3(4b + 12c + 11) \)

   c. \( 3(2x - 1) \)

   d. \( 10(b + 4c) \)
Lesson 4: Writing Products as Sums

Example 1: Distributive Property

For parts (a) and (b), apply the distributive property. Substitute the given numerical values to show the expressions are equivalent.

a. \(3(4f - 1), f = 2\)

b. \(9(-3r - 11), r = 10\)

c. Rewrite the expression, \((6x + 15) ÷ 3\), as a sum using the distributive property.

d. Expand the expression \(4(x + y + z)\).
Exercises

Rewrite the expressions as a sum.

a. \(2(2b + 12)\)

b. \(4(20r - 8)\)

c. \((49g - 7) \div 7\)

d. Expand the expression from a product to a sum so as to remove grouping symbols using an area model and the repeated use of distributive property: \(3(x + 2y + 5z)\).
Problem Set

1. Write each expression in standard form. Verify that your expressions are equivalent by testing \( x = 2 \).

   a. \(-3 (8x)\)
   
   b. \(-3 (8x + 6)\)
   
   c. \(8(5x + 3) + 2(3x)\)

2. Write each expression in standard form. Verify that your expressions are equivalent by testing \( x = 2 \).

   a. \(8x ÷ 2\)
   
   b. \(25x ÷ 5x\)
   
   c. \((6x + 2) ÷ 2\)
Lesson 5: Writing Sums as Products

Example 1: Factoring (Grinch Rule)
Rewrite the expressions as a product of two factors and verify that your expressions are equivalent by testing \( r = 3 \).

a. \( 3r + 6 \)  
b. \( 36r + 72 \)  
c. \( 55r + 11 \)  
d. \( 144r - 15 \)

Example 2
Find an equivalent expression by collecting like terms and factoring.

\( 4 + 5r - 9r \)

Exercises: Factor each of the following by collecting like terms and using the distributive property.

1. \( 8 - 12f + 8f \)

2. \( 3g + 9 - 10g + 5 \)

3. \( -5 - 8 + 11k + 12k \)
Example 3: Rational Number Factoring

Find an equivalent expression by factoring:

\[ \frac{1}{2}x + 6 \]

\[ \frac{4}{5}t + \frac{1}{5} \]

\[ 36p - 24 \]

\[ -\frac{3}{4}y - 2.5 \]
Problem Set

1. Use the following rectangular array to answer the questions below.

   a. Fill in the missing information.
   b. Write the sum represented in the rectangular array.
   c. Use the missing information from part (a) to write the sum from part (b) as a product of two factors.

   ![Rectangular Array]

2. Write the sum as a product of two factors.

   a. $81w + 48$

   b. $10 - 25t$

   c. $12a + 16b + 8$

   d. $-6c - 9$

   e. $\frac{1}{2}n + 8$

   f. $\frac{3}{4}t + \frac{1}{2}r - \frac{1}{4}s$
Lesson 6: Equivalent Expressions with Rational Numbers

Opening Exercise

Do the computations, leaving your answers in simplest/standard form. Show your steps.

1. Terry weighs 40 kg. Janice weighs $2\frac{3}{4}$ kg less than Terry. What is their combined weight?

2. $2 \frac{2}{3} - 1 \frac{1}{2} - \frac{5}{6}$

3. $4 \left(\frac{3}{5}\right)$

4. Mr. Jackson bought $1 \frac{3}{5}$ lbs of beef. He cooked $\frac{3}{4}$ of it for lunch. How much does he have left?

5. $\frac{2}{3}n + \frac{3}{4}n + \frac{1}{6}n + \frac{2}{9}n$
Example 1: Write the expressions in standard form by collecting like terms.

a. \( \frac{2}{5}g - \frac{1}{2} - g + \frac{3}{10}g - \frac{4}{5} \)

b. \( i + 6i - \frac{5}{6}i + \frac{1}{3}h + \frac{1}{2}i - h + \frac{1}{4}h \)

Example 2: Rewrite the expressions in standard form by finding the product and collecting like terms.

\(-6\frac{1}{3} - \frac{1}{2}(\frac{1}{2} + y)\)

Exercise 1: Rewrite the expression in standard form by finding the product and collecting like terms.

\( \frac{2}{3} + \frac{1}{3} \left( \frac{1}{4}f - 1 \frac{1}{3} \right) \)
Problem Set

1. Rewrite the expressions by collecting like terms.
   
   a. \( \frac{1}{2}k - \frac{3}{8}k \)
   
   b. \( \frac{2r}{5} + \frac{7r}{15} \)
   
   c. \( -\frac{1}{3}a - \frac{1}{2}b - \frac{3}{4} + \frac{1}{2}b - \frac{2}{3} + \frac{5}{6}a \)
   
   d. \( \frac{3}{5}q - \frac{1}{10}q + \frac{1}{9} - \frac{1}{9}p \)
   
   e. \( \frac{5}{7}y - \frac{y}{14} \)
   
   f. \( \frac{3n}{8} + \frac{n}{4} + \frac{n}{2} \)

2. Rewrite the expressions by using the distributive property and collecting like terms.
   
   a. \( \frac{4}{5}(15x - 5) \)
   
   b. \( \frac{4}{5}v - \frac{2}{3}(4v + \frac{1}{6}) \)
   
   c. \( \frac{1}{4}(p + 4) + \frac{3}{5}(p - 1) \)
   
   d. \( \frac{4}{5}(c - 1) - \frac{1}{8}(2c + 1) \)